Observation, Estimation et Commande robuste non linéaires d’actionneurs électriques sans capteur mécanique.

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Introduction
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   2.a Observer design for non linear systems
   2.b Interior Permanent Magnet Synchronous Motor synchronous motor case (IPMSM)
      - Model in (d-q) axes
      - Observer stability analysis
   2.c Induction Motor case (IM)
      - Induction motor model in a dq rotating frame
      - Adaptive Interconnected observer design
      - Stability analysis of observer
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   3.c IM case
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Introduction

Observation, Estimation et Commande robuste non linéaires d'actionneurs électriques sans capteur mécanique
Classical control scheme

$x$: Fluxes, currents and speed
$u$: Voltages
$T_l$: Load torque (disturbance)
$y$: Currents, speed and load torque
Classical control scheme

- Fluxes, currents and speed
- Voltages
- Load torque (disturbance)
- Currents, speed and load torque

- Cost,
- Breakdowns of sensor \(\Rightarrow\) maintenance cost,
- Fault tolerant control.
Classical control scheme

\[ \begin{align*}
x & : \text{ Fluxes, currents and speed} \\
u & : \text{ Voltages} \\
T_i & : \text{ Load torque (disturbance)} \\
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\end{align*} \]

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- Cost,
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Software sensor

Mechanical sensor replaced by Software sensor: Observer
Classical control scheme

\[ x: \text{Fluxes, currents and speed} \]
\[ u: \text{Voltages} \]
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Software sensor

Control without mechanical sensor + low speed $\Rightarrow$ problems
Classical control scheme

\( x \): Fluxes, currents and speed  
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\( T_l \): Load torque (disturbance)  
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- Cost,  
- Breakdowns of sensor \( \Rightarrow \) maintenance cost,  
- Fault tolerant control.

Control without mechanical sensor + low speed \( \Rightarrow \) problems

- Loss of observability,
**Classical control scheme**

- $x$: Fluxes, currents and speed
- $u$: Voltages
- $T_i$: Load torque (disturbance)
- $y$: Currents, speed and load torque
  - Cost,
  - Breakdowns of sensor $\Rightarrow$ maintenance cost,
  - Fault tolerant control.

**Software sensor**

- Mechanical sensor replaced by
  - Software sensor: Observer

**Control without mechanical sensor + low speed $\Rightarrow$ problems**

- Loss of observability,
- Robustness.
Industrial sensorless inverter experimental results

Test Load = 5Nm (50% nominal load)

a- speed ref (black).
b- speed measure (red)
c- inverter observed speed (blue)

Time (s)

Figure: 1. Test on the sensorless benchmark.
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5 Conclusions
Observer design for non linear systems (1)
No general constructive algorithm for a non linear system:

\[
\dot{x} = f(\ddot{x}) + g(\ddot{x})u, \\
y = h(\ddot{x}).
\] (1)
No general constructive algorithm for a non linear system:

\[
\dot{x} = f(x) + g(x)u, \\
y = h(x).
\] (1)

**A first solution:** to find a generalized transformation \( X = \phi(\tilde{x}, u, \hat{u}, \cdots, u^{(q-1)}) \) such that the nonlinear system (1) is equivalent to a linear one modulo an input-output (NL) injection:

\[
\begin{align*}
\dot{X} &= AX + \varphi(y, u, \hat{u}, \cdots, u^{(q)}), \\
\tilde{y} &= CX.
\end{align*}
\] (2)

Hypothesis: \( A \) et \( C \) are in observability canonical form. Then a observer (Luenberger Like) can be:

\[
\dot{\hat{X}} = A\hat{X} + \varphi(y, u, \hat{u}, \cdots, u^{(q)}) + K \cdot C \cdot (X - \hat{X})
\]

→ Exponentiel convergence tuned by gain \( K \),
→ Observation of system (1) given by \( \hat{x} = \phi^{-1}(\hat{X}, u, \hat{u}, \cdots, u^{(q-1)}) \),
→ See Souleiman, Glumineau, Guay, Respondek.
Observer design for non linear systems (2)
A second solution: to find a state transformation $X = \phi(\tilde{x}, u)$ such that the nonlinear system (1) is equivalent to an affine one modulo an input-output (NL) injection:

$$
\begin{align*}
\dot{X} &= A(y, u)X + \varphi(y, u) \\
\tilde{y} &= CX.
\end{align*}
$$

(3)

with some observability hypothesis (see Besançon and al.), an Kalman Like observer can be designed:

$$
\begin{align*}
\dot{\hat{X}} &= A(y, u)\hat{X} + \varphi(y, u) - S^{-1}C^T(C\hat{X} - \tilde{y}) \\
\dot{S} &= -\theta S - A(y, u)^T S - SA(y, u) + C^T C \\
\tilde{y} &= C\hat{X}.
\end{align*}
$$

(4)

There exists $\theta > 0$ such that system (4) is an observer for (3) and 

$$
||\dot{\hat{X}} - X|| \leq \lambda \exp(-\tau t), \forall \tau > 0.
$$

→ Observation of system (1) is given by $\hat{x} = \phi^{-1}(\hat{X}, u, \dot{u}, \cdots, u^{(q-1)})$. → See Souleiman, Glumineau, Besancon
A second solution: to find a state transformation $X = \phi(\tilde{x}, u)$ such that the nonlinear system (1) is equivalent to an affine one modulo an input-output (NL) injection:

$$\begin{align*}
\dot{X} &= A(y, u)X + \varphi(y, u) \\
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\end{align*}$$

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\dot{X} &= A(y, u)\dot{X} + \varphi(y, u) - S^{-1}C^T(C\dot{X} - \ddot{y}) \\
\dot{S} &= -\theta S - A(y, u)^T S - SA(y, u) + C^T C \\
\dot{y} &= C\dot{X}
\end{align*}$$

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There exists $\theta > 0$ such that system (4) is an observer for (3) and

$$||\dot{X} - X|| \leq \lambda \exp(-\tau t), \forall \tau > 0.$$ 

→ Observation of system (1) is given by $\hat{x} = \phi^{-1}(\dot{X}, u, \dot{u}, \cdots, u^{(q-1)})$. → See Souleiman, Glumineau, Besançon

Otherwise ... Other observers can be designed with Kalman Like High gain observer, Interconnected observers, ... (see Besançon, Hammouri ...)

Sensorless problem
Sensorless problem

**Means**

- Design of an adaptive interconnected high-gain observers for the estimation of the magnetic (fluxes), mechanical (speed, load torque) variables, stator resistance value, ....
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- Design of an adaptive interconnected high-gain observers for the estimation of the magnetic (fluxes), mechanical (speed, load torque) variables, stator resistance value, ....
- Design of High Order Sliding Mode Controllers to achieve speed and flux tracking (IM) or speed and optimum torque (IPMSM) with robust performances
Sensorless problem

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- Design of an adaptive interconnected high-gain observers for the estimation of the magnetic(fluxes), mechanical (speed, load torque) variables, stator resistance value, ....

- Design of High Order Sliding Mode Controllers to achieve speed and flux tracking (IM) or speed and optimum torque (IPMSM) with robust performances

- Analysis of the stability of the Schemes.
Sensorless problem

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- Design of an adaptive interconnected high-gain observers for the estimation of the magnetic (fluxes), mechanical (speed, load torque) variables, stator resistance value, ...
- Design of High Order Sliding Mode Controllers to achieve speed and flux tracking (IM) or speed and optimum torque (IPMSM) with robust performances
- Analysis of the stability of the Schemes.
- Validation on “Sensorless Control Benchmarks” from french research group of the CNRS Control of Electric Systems, http://www2.irccyn.ec-nantes.fr/CSE/.
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Nonlinear model of the IPM synchronous motor in the $dq$ frame

From the PARK et CONCORDIA transformations:

$$\Sigma_{NL} : \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases}$$  \hspace{1cm} (5)

with

$$x = (i_d, i_q, \Omega, \theta)^T, u = (u_d, u_q)^T, y = (h_1, h_2)^T = (i_d, i_q)^T$$

$$f(x) = \begin{pmatrix} \frac{R_s}{L_d} i_d + \frac{p}{L_q} \Omega i_q \\ -\frac{R_s}{L_q} i_q - \frac{p}{L_q} \Omega i_d - \frac{1}{L_q} \phi_f \Omega s \\ \frac{p}{J} (L_d - L_q)i_d i_q - \frac{f_v}{J} \Omega + \frac{p}{J} \phi_f i_q - \frac{1}{J} T_l \end{pmatrix}$$

and

$$g(x) = \begin{pmatrix} \frac{1}{L_d} \\ 0 \\ 0 \end{pmatrix}$$

with $i_d, i_q, u_d, u_q$: $dq$ currents, voltages and $\Omega$, $T_l$, $R_s$, $\theta$: motor speed, load torque, stator resistance, rotor position.
Nonlinear model + uncertainties of the IPMSM in the synchronous frame (d-q)

or

\[
\begin{align*}
\dot{i}_d &= k_4 i_d + k_5 \Omega i_q + k_6 v_d \\
\dot{i}_q &= k_9 i_q + k_8 \Omega i_d + k_7 \Omega + k_{10} v_q \\
\dot{\Omega} &= k_1 i_d i_q + k_3 \Omega + k_2 i_q \\
\dot{\theta} &= \Omega 
\end{align*}
\]  
(6)

with

Parameter uncertainties

\[
\begin{align*}
k_1 &= k_{01} + \delta k_1 = \frac{p}{J} (L_d - L_q), \\
k_2 &= k_{02} + \delta k_2 = \frac{p \phi_f}{J}, \\
k_3 &= k_{03} + \delta k_3 = -\frac{f_v}{J}, \\
k_4 &= k_{04} + \delta k_4 = -\frac{R_s}{L_d}, \\
k_5 &= k_{05} + \delta k_5 = p \frac{L_q}{L_d}, \\
k_6 &= k_{06} + \delta k_6 = \frac{1}{L_d}, \\
k_7 &= k_{07} + \delta k_7 = -\frac{p \phi_f}{L_q}, \\
k_8 &= k_{08} + \delta k_8 = -\frac{p L_d}{L_q}, \\
k_9 &= k_{09} + \delta k_9 = -\frac{R_s}{L_q}, \\
k_{10} &= k_{010} + \delta k_{10} = \frac{1}{L_q}. 
\end{align*}
\]  
(7)

\(k_{0i} (1 \leq i \leq 10)\) the nominal value of the concerned parameter, \(\delta k_i\) the uncertainty on this parameter such that

\[|\delta k_i| \leq \delta k_{0i} < |k_{0i}|\]

with \(\delta k_{0i}\) a known positive bound.
Interconnected Model

The IPMSM model (1) can be written in an interconnected form:

\[
\begin{align*}
\Sigma_1 & \quad \begin{cases} 
\dot{X}_1 &= A_1(y)X_1 + g_1(X_2, u) \\
y_1 &= C_1 X_1 
\end{cases} \\
\Sigma_2 & \quad \begin{cases} 
\dot{X}_2 &= A_2(y)X_2 + g_2(X_1, X_2, u) + \Phi T_l \\
y_2 &= C_2 X_2 
\end{cases}
\end{align*}
\]

(8)
(9)

\(T_l\) and \(R_s\) are assumed to be piecewise functions of time.
Interconnected Model

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\end{cases} \tag{8}
\end{align*}
\]

\[
\begin{align*}
\Sigma_2: \begin{cases}
\dot{X}_2 &= A_2(y)X_2 + g_2(X_1, X_2, u) + \Phi T_l \\
y_2 &= C_2 X_2 
\end{cases} \tag{9}
\end{align*}
\]

\(T_l\) and \(R_s\) are assumed to be piecewise functions of time.

where

\[
X_1 = \begin{bmatrix} i_q \\ R_s \end{bmatrix}, X_2 = \begin{bmatrix} i_d \\ \Omega \end{bmatrix}, A_1(\cdot) = \begin{bmatrix} 0 & -i_q/L_q \\ 0 & 0 \end{bmatrix}, A_2(\cdot) = \begin{bmatrix} 0 & pLq i_q \\ 0 & -f_v \end{bmatrix}
\]

\[
g_1(\cdot) = \begin{bmatrix} -pLd \Omega i_d - p\phi_f/L_q \Omega + 1/L_q u_q \\ 0 \end{bmatrix}, g_2(\cdot) = \begin{bmatrix} -R_s i_d + 1/L_d u_d \\ p/J \Phi_f i_q i_d + p/J (L_d - L_q) i_q \end{bmatrix},
\]

\[
\Phi = \begin{bmatrix} 0 & 0 \\ 1 & -J \end{bmatrix}, C_1 = C_2 = [1 \ 0].
\]
Objectives

Design two adaptive interconnected observers for $\Sigma_1$ and $\Sigma_2$ to estimate the speed the load torque and the stator resistance.
## Design (1)

### Objectives

Design two adaptive interconnected observers for $\Sigma_1$ and $\Sigma_2$ to estimate the speed, the load torque and the stator resistance.

### Assumption 1

$(u, X_2)$ and $(u, X_1)$ are respectively regularly persistent inputs for $\Sigma_1$ and $\Sigma_2$. (i.e. the inputs do not force the system to stay in an observability loss area)
Design (1)

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Remark 1.

$X_1$ and $X_2$ are respectively considered as inputs for subsystem $\Sigma_2$ and $\Sigma_1$. 
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Design two adaptive interconnected observers for $\Sigma_1$ and $\Sigma_2$ to estimate the speed the load torque and the stator resistance.

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$(u, X_2)$ and $(u, X_1)$ are respectively regularly persistent inputs for $\Sigma_1$ and $\Sigma_2$. (i.e. the inputs do not force the system to stay in an observability loss area)

Remark 1.

$X_1$ and $X_2$ are respectively considered as inputs for subsystem $\Sigma_2$ and $\Sigma_1$.

Remark 2.

To design robust adaptive observers, it will be necessary to verify (easy to do):

$\rightarrow A_1(y)$ is globally Lipschitz $w.r.t. X_1$.
$\rightarrow A_2(y)$ is globally Lipschitz $w.r.t. X_1$.
$\rightarrow g_1(X_2)$ is globally Lipschitz $w.r.t. X_2$ uniformly $w.r.t. (u, y)$.
$\rightarrow g_2(X_1, X_2)$ is globally Lipschitz $w.r.t. X_1, X_2$ uniformly $w.r.t. (u, y)$.
Observers

Then adaptive interconnected observers of $\Sigma_1$ and $\Sigma_2$ are given by:

\[
\begin{align*}
\dot{Z}_1 &= A_1(y)Z_1 + g_1(y_2, Z_2, u) + S_1^{-1}C_1^T(y_1 - \hat{y}_1) \\
\dot{S}_1 &= -\rho_1 S_1 - A_1^T(y)S_1 - S_1A_1(y) + C_1^TC_1 \\
\hat{y}_1 &= C_1Z_1 \\
\dot{Z}_2 &= A_2(y)Z_2 + g_2(y, Z_1, u) + \Phi \hat{T}_l + KC_1^T(y_1 - \hat{y}_1) \\
&\quad + (\omega \Lambda S_\theta^{-1} \Lambda^T C_2^T + \Gamma S_x^{-1} C_2^T)(y_2 - \hat{y}_2) \\
\dot{\hat{T}}_l &= \omega S_\theta^{-1} \Lambda^T C_2^T(y_2 - \hat{y}_2) + B(Z_1)(y_1 - \hat{y}_1) \\
\dot{S}_x &= -\rho_x S_x - A_2^T(Z_1, y)S_x - S_xA_2(Z_1, y) + C_2^TC_2 \\
\dot{S}_\theta &= -\rho_\theta S_\theta + \Lambda^T C_2^TC_2\Lambda \\
\dot{\Lambda} &= (A_2(y) - \Gamma S_x^{-1} C_2^TC_2)\Lambda + \Phi \\
\hat{y}_2 &= C_2Z_2
\end{align*}
\]

with $Z_1 = \begin{bmatrix} \hat{i}_q & \hat{R}_s \end{bmatrix}^T$ and $Z_2 = \begin{bmatrix} \hat{i}_d & \hat{\Omega} \end{bmatrix}^T$ are the estimated states of $X_1$ and $X_2$. $S_1^{-1}C_1^T$ is the gain of the observer (10), $\omega \Lambda S_\theta^{-1} \Lambda^T C_2^T + \Gamma S_x^{-1} C_2^T$ and $KC_1^T$ are the gains of the observer (11).
Observer Stability

Let us define:

\[ \epsilon_1 = X_1 - Z_1, \quad \epsilon_2 = X_2 - Z_2, \quad \epsilon_3 = T_I - \hat{T}_I. \]

We take the following Lyapunov candidate function

\[ V_o = V_1 + V_2 + V_3 \]

with \( V_1 = \epsilon_1^T S_1 \epsilon_1, \) \( V_2 = \epsilon_2^T S_2 \epsilon_2 \) et \( V_3 = \epsilon_3^T S_3 \epsilon_3. \)

The time derivative of \( V_o \) is:

\[
\dot{V}_o \leq -\delta(V_1 + V_2 + V_3) + \mu(\sqrt{V_1} + \sqrt{V_2}).
\]

\[
\leq -\delta V_o + \mu \psi \sqrt{V_o},
\]

where \( \psi > 0 \), such that \( \psi \sqrt{V_1 + V_2 + V_3} > \sqrt{V_1} + \sqrt{V_2} \) and \( \delta = \min(\delta_1, \delta_2, \delta_3) \) with

\[
\delta_1 = \rho_1 - \bar{\lambda}_1 \psi_1 > 0, \quad \delta_2 = \rho_2 - \bar{\lambda}_2 \psi_2 - \frac{\bar{\lambda}_1}{\psi_1} - \bar{\lambda}_3 > 0, \quad \delta_3 = \rho_n - \frac{\bar{\lambda}_2}{\psi_2} > 0
\]

and \( \mu \) an uncertainties parameter.
Discussion

- No parameters variation (i.e. nominal case) $\Rightarrow \mu = 0$, by choosing $\rho_1$, $\rho_2$ and $\rho_3$ such that $\delta_1 > 0$, $\delta_2 > 0$ and $\delta_3 > 0$ implies $\dot{V}_o \leq -\delta V_o$,
Discussion

- No parameters variation (i.e. nominal case) ⇒ $\mu = 0$, by choosing $\rho_1$, $\rho_2$ and $\rho_3$ such that $\delta_1 > 0$, $\delta_2 > 0$ and $\delta_3 > 0$ implies $\dot{V}_o \leq -\delta V_o$.

- Parameters variation ⇒ $\mu \neq 0$ then $\dot{V}_o \leq -(1 - \varsigma)\delta V_o$, $\forall \|\epsilon\| \geq \epsilon^*$ such that $B(0, \epsilon^*)$ is a ball which the radius depends of the uncertainties parameter $\mu$ and of the gains of the observer.
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5 Conclusions
Nonlinear model of the induction motor (dq frame)

From Park and Concorda transformations, one gets

\[ \dot{x} = f(x) + g(x)u \]
\[ y = h(x) \]  \hspace{1cm} (13)

with

\[ x = (i_{sd}, i_{sq}, \phi_{rd}, \phi_{rq}, \Omega, T_l, R_s) \]
\[ u = (u_{sd}, u_{sq}) \]
\[ y = (y_1, y_2) \]

\[ f(x) = \begin{pmatrix}
  ba\phi_{rd} + bp\Omega\phi_{rq} - \gamma i_{sd} + \omega s i_{sq} \\
  ba\phi_{rq} - bp\Omega\phi_{rd} - \gamma i_{sq} - \omega s i_{sd} \\
  -a\phi_{rd} + (\omega_s - p\Omega)\phi_{rq} + aM_{sr}i_{sd} \\
  -a\phi_{rq} - (\omega_s - p\Omega)\phi_{rd} + aM_{sr}i_{sq} \\
  m(\phi_{rd}i_{sq} - \phi_{rq}i_{sd}) - c\Omega - \frac{i}{j}T_l \\
  0 \\
  0
\end{pmatrix} \]

and

\[ g(x) = \begin{pmatrix}
  m_1 & 0 \\
  0 & m_1 \\
  0 & 0 \\
  0 & 0 \\
  0 & 0 \\
  0 & 0
\end{pmatrix}. \]

with \( i_{sd}, i_{sq}, \phi_{rd}, \phi_{rq}, u_{sd}, u_{sq}: \) dq currents, flux, voltage, \( \Omega, T_l, \omega_s, R_s: \) motor speed, load torque, stator pulsation, stator resistance.
Interconnected observers: principle [Besançon, 98], [Besançon, 06]
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A class of nonlinear systems may be seen as the interconnection between several subsystems, such that each of these subsystems satisfies some required properties to build an observer.
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The main idea is to design adaptive interconnected observers for the whole system, from the separate synthesis of an observer for each subsystem, assuming that, for each of these separate observer, the state of the other subsystem is available.
Design (1)

The IM model (13) can be written in an interconnection form

$$\Sigma_1 : \begin{cases} \dot{X}_1 = A_1(X_2, y)X_1 + g_1(u, y, X_2, X_1) + \Phi T_l \\ y_1 = C_1 X_1 \end{cases}$$

$$\Sigma_2 : \begin{cases} \dot{X}_2 = A_2(X_1)X_2 + g_2(u, y, X_1, X_2) \\ y_2 = C_2 X_2 \end{cases}$$

$$X_1 = (i_{sd}, \Omega, R_s)^T$$, $$X_2 = (i_{sq}, \phi_{rd}, \phi_{rq})^T$$, $$u = (u_{sd}, u_{sq})^T$$, $$y = (i_{sd}, i_{sq})^T$$,
$$C_1 = C_2 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$.

$T_l$ is the unknown load torque supposed a piecewise constant function. $R_s$ is the unknown stator resistance value supposed constant.

$$A_1(\cdot) = \begin{pmatrix} 0 & bp_{\phi_{rq}} & -m_1 i_{sd} \\ -m_{\phi_{rq}} & -c & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad A_2(\cdot) = \begin{pmatrix} -\gamma_1 & -bp\Omega & ab \\ 0 & -a & -p\Omega \\ 0 & p\Omega & -a \end{pmatrix},$$

$$g_1(\cdot) = \begin{pmatrix} -\gamma_1 i_{sd} + ab\phi_{rd} + m_1 u_{sd} + \omega_s i_{sq} \\ m\phi_{rd} i_{sq} \\ 0 \end{pmatrix},$$

$$g_2(\cdot) = \begin{pmatrix} -m_1 R_s i_{sq} - \omega_s i_{sd} + m_1 u_{sq} \\ \omega_s \phi_{rq} + aM_{sr} i_{sd} \\ -\omega_s \phi_{rd} + aM_{sr} i_{sq} \end{pmatrix}, \quad \Phi = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}.$$
Design (2)

Objectives

Design of two adaptive interconnected high-gains observers for $\Sigma_1$ and $\Sigma_2$ in order to estimate mechanical variables (speed, load torque) and stator resistance value and magnetic variables (fluxes)
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1. $X_1$ and $X_2$ are respectively considered as inputs for subsystem $\Sigma_2$ and $\Sigma_1$.
2. $(u, X_2)$ and $(u, X_1)$ are respectively regularly persistent inputs for $\Sigma_1$ and $\Sigma_2$. 
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$g_1(u, y, X_2, X_1)$ is globally Lipschitz with respect to $X_2, X_1$ and uniformly w.r.t. $(u, y)$. 
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$g_1(u, y, X_2, X_1)$ is globally Lipschitz with respect to $X_2$, $X_1$ and uniformly w.r.t. $(u, y)$.

Remark

It is easy to verify that $A_1(X_2, y)$ is globally Lipschitz w.r.t. $X_2$, $A_2(X_1)$ is globally Lipschitz w.r.t. $X_1$ and $g_2(u, y, X_2, X_1)$ is globally Lipschitz w.r.t. $X_2$, $X_1$ and uniformly w.r.t. $(u, y)$.
Then, the adaptive interconnected observers of $\Sigma_1$ and $\Sigma_2$ read as

$$
\dot{Z}_1 = A_1(Z_2, y)Z_1 + g_1(u, y, Z_2, Z_1) + \Phi \hat{T}_l \\
+ (\omega \Lambda S_3^{-1} \Lambda^T C_1^T + \Gamma S_1^{-1} C_1^T)(y_1 - \hat{y}_1) + KC_2^T (y_2 - \hat{y}_2) \\
\dot{T}_l = \omega S_3^{-1} \Lambda^T C_1^T (y_1 - \hat{y}_1) + B_1(Z_2)(y_2 - \hat{y}_2) + B_2(Z_2)(y_1 - \hat{y}_1) \\
\dot{S}_1 = -\theta_1 S_1 - A_1^T(Z_2, y)S_1 - S_1 A_1(Z_2, y) + C_1^T C_1 \\
\dot{S}_3 = -\theta_3 S_3 + \Lambda^T C_1^T C_1 \Lambda \\
\dot{\Lambda} = (A_1(Z_2, y) - \Gamma S_1^{-1} C_1^T C_1) \Lambda + \Phi \\
\hat{y}_1 = C_1 Z_1
$$

$$
\dot{Z}_2 = A_2(Z_1)Z_2 + g_2(u, y, Z_1, Z_2) + S_2^{-1} C_2^T (y_2 - \hat{y}_2) \\
\dot{S}_2 = -\theta_2 S_2 - A_2^T(Z_1)S_2 - S_2 A_2(Z_1) + C_2^T C_2 \\
\hat{y}_2 = C_2 Z_2
$$
Design (4)

Definitions

\[ Z_1 = (i_{sd}, \hat{\Omega}, \hat{R}_s)^T, \quad Z_2 = (i_{sq}, \hat{\phi}_{rd}, \hat{\phi}_{rq})^T \]
are the estimated states. \( \theta_1, \theta_2, \theta_3 \) are positive constants and \( S_1, S_2 \) are symmetric positive definite matrices \([\text{Besançon, 96}], S_3(0) > 0\).

\[ B_1(Z_2) = k m \hat{\phi}_{rd}, \quad B_2(Z_2) = -k m \hat{\phi}_{rq}, \quad K = \begin{pmatrix} -k_{c1} & 0 & 0 \\ -k_{c2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \alpha \end{pmatrix}, \]
where \( k, k_{c1}, k_{c2}, \alpha \) and \( \varpi \) are positive constants.

Note that \((\varpi \Lambda S_3^{-1} \Lambda^T C_1^T + \Gamma S_1^{-1} C_1^T)\) and \(KC_2^T\) are the gains of observer \((O_1)\) and \(S_2^{-1} C_2^T\) is the gain of observer \((O_2)\).
Observer Stability analysis under parameters uncertainties (and tuning), [Traore, 07], [Besançon, 06]

Observer Stability and Tuning

Let us define $V_o = V_1 + V_2 + V_3$ a candidate Lyapunov function where $V_1 = \epsilon_1^T S_1 \epsilon_1$, $V_2 = \epsilon_2^T S_2 \epsilon_2$ and $V_3 = \epsilon_3^T S_3 \epsilon_3$.

with $\epsilon_1$, $\epsilon_2$ and $\epsilon_3$ are the errors between the state and its estimation.

The time derivative of $V_o$ is:

$$
\dot{V}_o \leq -\delta(V_1 + V_2 + V_3) + \mu(\sqrt{V_1} + \sqrt{V_2})
\leq -\delta V_o + \mu \psi \sqrt{V_o}.
$$

(16)

where $\psi > 0$, such that $\psi \sqrt{V_1 + V_2 + V_3} > \sqrt{V_1} + \sqrt{V_2}$.

$\delta = \min(\delta_1, \delta_2, \delta_3)$ and $\mu = \max(\mu_6, \mu_7)$ parameters uncertainties,

$\delta_1 = \theta_1 - 2k_{12} - 2k_1k_{16} - \tilde{\mu}_1 \varphi_1 - \tilde{\mu}_9 \varphi_3 > 0$, $\delta_3 = \theta_3 + 2k_{10} - \frac{\tilde{\mu}_8}{\varphi_2} - \frac{\tilde{\mu}_9}{\varphi_3} > 0$, and

$\varphi_i (i = 1, 2, 3) \in ]0, 1[$
Discussion
Discussion

- Known parameters $\Rightarrow \mu = 0$, to choose $\theta_1$, $\theta_2$ and $\theta_3$ such that $\delta_1 > 0$, $\delta_2 > 0$ and $\delta_3 > 0$ then $\dot{V}_o \leq -\delta V_o$. 


Discussion

- Known parameters $\Rightarrow \mu = 0$, to choose $\theta_1$, $\theta_2$ and $\theta_3$ such that $\delta_1 > 0$, $\delta_2 > 0$ and $\delta_3 > 0$ then $\dot{V}_{o} \leq -\delta V_{o}$.

- Parameters variation $\Rightarrow \mu \neq 0$ then $\dot{V}_{o} \leq -(1 - \varsigma)\delta V_{o}, \quad \forall \|\epsilon\| \geq \frac{\mu \psi}{\varsigma \delta}, 1 > \varsigma > 0$
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Hypothesis and Definition HOSM (1)

Consider the uncertain nonlinear system

\[
\begin{align*}
\dot{x} &= f(x) + g(x)v \\
y &= \sigma(x, t)
\end{align*}
\]  

(17)  

(18)

with \(x\) the state variable, \(v\) the input control and \(\sigma(x, t)\) a smooth output function (sliding variable) defined to satisfy the control objective (tracking trajectory, ...).

**H1.** The relative degree \(r_d\) of (17) with respect to \(s\) is assumed to be constant and known, and the associated zero dynamics are stable.

After a possible preliminary I/O linearization feedback (computed with the nominal parameters) applied to the nonlinear system (17) there exists \(\chi\) and \(\Gamma\) such that

\[
\sigma^{(r_d)} = \chi(\cdot) + \Gamma(\cdot)v
\]

(19)

**H2.** Functions \(\chi(\cdot)\) and \(\Gamma(\cdot)\) are such that there exist \(K_m \in \mathbb{R}^{++}, K_M \in \mathbb{R}^{++}, C_0 \in \mathbb{R}^{+}\) with

\[
|\chi| \leq C_0, \quad 0 < K_m < \Gamma < K_M.
\]

Furthermore, control input \(v\) is bounded.

(20)
HOSM controller design (1)
The design of the controller is obtained in two steps
HOSM controller design (1)

The design of the controller is obtained in two steps:
- Design of a switching variable $S(\sigma, \dot{\sigma}, \ldots)$
HOSM controller design (1)

The design of the controller is obtained in two steps

- Design of a switching variable $S(\sigma, \dot{\sigma}, ...)$
- Design of a control to maintain, from $t = 0$, the system trajectories on the corresponding switching surface which ensures the establishment of a $r^{th}$ order sliding mode, in spite of uncertainties in a fixed finite time $t_F$.

$\implies$ for $t > t_F$, $\sigma = 0$, $\dot{\sigma} = 0$, ..., $\sigma^{(r)} = 0$, with $r \geq$ the relative degree $r_d$. 
HOSM controller design (2)

**Switching variable.** $S$ denotes the switching variable defined as

$$S = \sigma^{(r-1)} - \mathcal{F}^{(r-1)}(t) + \lambda_{r-2} [\sigma^{(r-2)} - \mathcal{F}^{(r-2)}(t)] + \cdots + \lambda_0 [\sigma(x, t) - \mathcal{F}(t)]$$

(21)

with
S denotes the switching variable defined as

\[ S = \sigma^{(r-1)} - \mathcal{F}^{(r-1)}(t) + \lambda_{r-2} [\sigma^{(r-2)} - \mathcal{F}^{(r-2)}(t)] + \cdots + \lambda_0 [\sigma(x, t) - \mathcal{F}(t)] \]  

(21)

with

\[ \lambda_{r-2}, \ldots, \lambda_0 \] defined such that \( P(z) = z^{(r-1)} + \lambda_{r-2} z^{(r-2)} + \cdots + \lambda_0 \) is a Hurwitz polynomial in the complex variable \( z \),
Switching variable. $S$ denotes the switching variable defined as

$$S = \sigma^{(r-1)} - \mathcal{F}^{(r-1)}(t) + \lambda_{r-2} \left[ \sigma^{(r-2)} - \mathcal{F}^{(r-2)}(t) \right] + \cdots + \lambda_0 \left[ \sigma(x, t) - \mathcal{F}(t) \right]$$

(21)

with

- $\lambda_{r-2}, \cdots, \lambda_0$ defined such that $P(z) = z^{(r-1)} + \lambda_{r-2}z^{(r-2)} + \cdots + \lambda_0$ is a Hurwitz polynomial in the complex variable $z$,
- $\mathcal{F}(t)$ is a $C^r$ function defined such that
  $$\sigma^{(k)}(x(0), 0) - \mathcal{F}^{(k)}(0) = 0$$
  and
  $$\sigma^{(k)}(x(t_F), t_F) - \mathcal{F}^{(k)}(t_F) = 0$$

$(0 \leq k \leq r - 1)$, $t_F$ the desired convergence time.
More precisely, the problem consists in finding the function $F(t)$ such that from initial and final conditions

$$
\begin{align*}
\sigma(x(0), 0) &= F(0), \\
\dot{\sigma}(x(0), 0) &= \dot{F}(0), \\
\sigma(x(t_F), t_F) &= F(t_F) = 0, \\
\dot{\sigma}(x(t_F), t_F) &= \dot{F}(t_F) = 0, \\
\vdots \\
\sigma^{(r-1)}(x(0), 0) &= F^{(r-1)}(0), \\
\sigma^{(r-1)}(x(t_F), t_F) &= F^{(r-1)}(t_F) = 0
\end{align*}
$$

A solution for $F(t)$ reads as $(1 \leq j \leq r)$

$$
F(t) = Ke^{FT}T\sigma^{(r-j)}(0)
$$

with $F$ a $2r \times 2r$-dimensional Hurwitz matrix (strictly negative eigenvalues), $T$ a $2r \times 1$-dimensional vector and $j$ such that $\sigma^{(r-j)}(0) \neq 0$ and bounded.
Then, the switching variable $S$ (7) reads as

$$
S = \sigma^{(r-1)} - KF^{r-1}e^{Ft}T\sigma^{(r-j)}(0) + \lambda_{r-2} [\sigma^{(r-2)} - KF^{r-2}e^{Ft}T\sigma^{(r-j)}(0)] \\
+ \cdots + \lambda_0 [\sigma(x, t) - Ke^{Ft}T\sigma^{(r-j)}(0)] .
$$

(24)

**H3.** There exists a finite positive constant $\Theta \in R^+$ such that

$$
|KF^r e^{Ft} T\sigma^{(r-j)}(0) - \lambda_{r-2} [\sigma^{(r-1)} - KF^{r-1}e^{Ft}T\sigma^{(r-j)}(0)] \\
- \cdots - \lambda_0 [\dot{\sigma}(x, t) - KFe^{Ft}T\sigma^{(r-j)}(0)]| < \Theta
$$

(25)

Equation $S = 0$ describes the desired dynamics which satisfy the finite time stabilization of vector $[\sigma^{(r-1)} \sigma^{(r-2)} \cdots \sigma]^T$ to zero.

Then, the *switching manifold* on which system is forced to slide on, via a discontinuous control $\nu$, is defined as

$$
S = \{x \mid S = 0\}
$$

(26)
HOSM controller design (5)

The discontinuous control law \( v \) must force the system trajectories to slide on \( S \), to reach in finite time the origin and to maintain the system at the origin.

**Theorem** Consider nonlinear system (17) with a relative degree \( r_d \) with respect to \( \sigma(x, t) \).

Suppose that it is minimum phase and that hypotheses \( H_1 \) and \( H_2 \) are fulfilled. Let \( r \) be the sliding mode order and \( 0 < t_F < \infty \) the desired convergence time. Define \( S \in R \) by (24) with \( K \) unique solution of (23) and that assumption H3 is fulfilled. The control input \( v \) defined by

\[
\nu = -\alpha \text{ sign}(S)
\]

with

\[
\alpha \geq \frac{C_0 + \Theta + \eta}{K_m},
\]

\( C_0, K_m \) defined by (20), \( \Theta \) defined by (25), \( \eta > 0 \), leads to the establishment of a \( r^{th} \) order sliding mode with respect to \( \sigma \). The convergence time is \( t_F \).
SM1 and this HOSM

What is the meaning? (illustrated for order $r = 2$)

**SM of order 1**

$$S = \dot{\sigma} + \frac{1}{T} \sigma$$

**SM of order 2**

$$S = \dot{\sigma} + \mathcal{F}$$

$$\dot{\sigma} = S - \mathcal{F}$$

$\Rightarrow$ finite time convergence

Figure: 2. Comparaison of SM of order 1 and this HOSM design.
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5 Conclusions
The control objective is double:

- $\Omega \to \Omega_{\text{ref}}$
- $i_d \to i_{\text{dref}}$ where $i_{\text{dref}}$ is computed by the Maximum-Torque-Per-Ampere method [Uddin et al. 2007] to increase the efficiency of the drive.

HOSM controller design (1)

The design of the controller is given by:

- The choice of sliding variables for system (2), to obtain trajectory tracking to $\Omega_{\text{ref}}$ and $i_{\text{dref}}$. The **sliding variables** are chosen as:

  $$\sigma = \begin{bmatrix} \sigma_{\Omega} \\ \sigma_{i_d} \end{bmatrix} = \begin{bmatrix} \Omega - \Omega_{\text{ref}}(t) \\ i_d - i_{\text{dref}}(t) \end{bmatrix} \quad (29)$$

  The vector relative degree of $\sigma = [2 \ 1]^T$.

- Design of a control to maintain, from $t = 0$, the system trajectories on a switching manifold which ensures the establishment of a $r^{th}$ order sliding mode, in spite of uncertainties in a fixed finite time $t_F$. $\implies$ for $t > t_F$, $\sigma = 0$, $\dot{\sigma} = 0$, ..., $\sigma^{(r)} = 0$, with $r \geq$ the relative degree.

- As the discontinuous control introduced in the following, could imply a chattering phenomenon, the controller is chosen as a $[3 - 2]$ order sliding mode. $\implies$ The discontinuity control part will act on the first time derivative of the control inputs.
HOSM controller design (2)

The second and first time derivatives of $\sigma_\Omega$ and $\sigma_{id}$ are:

$$
\sigma^{(2)}_\Omega = \chi_1 + \Gamma_{11} u_d + \Gamma_{12} u_q, \quad \sigma^{(1)}_{id} = \chi_2 + \Gamma_{21} u_q. 
$$

(30)

Applying the following static state feedback

$$
\begin{bmatrix}
    u_d \\
    u_q
\end{bmatrix} = \Gamma_0^{-1} \cdot [-\chi_0 + \nu]
$$

(31)

denoting

$$
\Gamma_0 = \begin{bmatrix}
    \Gamma_{110} & \Gamma_{120} \\
    \Gamma_{210} & 0
\end{bmatrix}, \quad \chi_0 = \begin{bmatrix}
    \chi_{10} \\
    \chi_{20}
\end{bmatrix}
$$

(32)

with $\Gamma_0, \chi_0$ the nominal values terms and $\nu := [v_d \ v_q]^T$ the new control vector, then

$$
\begin{bmatrix}
    \sigma^{(3)}_\Omega \\
    \sigma^{(2)}_{id}
\end{bmatrix} = \begin{bmatrix}
    \ddot{\chi}_1 \\
    \ddot{\chi}_2
\end{bmatrix} + \begin{bmatrix}
    \dddot{\Gamma}_{11} & \dddot{\Gamma}_{12} \\
    \dddot{\Gamma}_{21} & 0
\end{bmatrix} \begin{bmatrix}
    \dot{v}_d \\
    \dot{v}_q
\end{bmatrix}
$$

(33)

with $\dot{v}_d, \dot{v}_q$ the "new inputs" defined later.
HOSM controller design (3) Switching vector

Then, following [Plestan et al. 2008], the switching vector $S = [S_\Omega \ S_{id}]^T$ is written as

$$
S_\Omega = \sigma_\Omega^{(2)} - \mathcal{F}_1^{(2)} + \lambda_{11}[\dot{\sigma}_\Omega - \dot{\mathcal{F}}_1] + \lambda_{12}[\sigma_\Omega - \mathcal{F}_1] \\
S_{id} = \sigma_{id}^{(1)} - \dot{\mathcal{F}}_2 + \lambda_{21}[\sigma_{id} - \mathcal{F}_2]
$$

with

$$
\mathcal{F}_1(t) = K_1 F_1 e^{F_1 t} T_1 \sigma_\Omega(0) \quad \text{et} \quad \mathcal{F}_2(t) = K_2 e^{F_2 t} T_2 \sigma_{id}(0).
$$

and $\lambda_{11} = 2\zeta_\Omega \omega_{n\Omega}$, $\lambda_{12} = 2\omega^2_{n\Omega}$, $\lambda_{21} = \omega_{nd}$

with $F_i$ being a $2r \times 2r$-dimensional stable matrix, and $T_i$ being a $2r \times 1$-dimensional vector.

$\implies$ Finite time convergence ($t_F$) to $\sigma = 0$, $\dot{\sigma} = 0$, ..., $\sigma^{(r)} = 0$. 

Observation, Estimation et Commande robuste non linéaire
HOSM controller design (3) Discontinuous input

From these switching surfaces (9), the discontinuous control input acting the "new" inputs is defined by:

\[
\begin{bmatrix}
\dot{v}_d \\
\dot{v}_q
\end{bmatrix}
= 
\begin{bmatrix}
-\alpha_1 \text{sign}(S_{\Omega}) \\
-\alpha_2 \text{sign}(S_{i_d})
\end{bmatrix}.
\]  (35)

where

\[
\alpha_i \geq \frac{C_{0i} + \Theta_i + \eta_i}{K_{mij}}
\]  (36)

with \( \Theta_i \) such that ([Plestan et al, 2008], [Traore et al, 2008]):

\[
\Theta > |K_c T F^r e^{Ft} \sigma_c^{(r-j)}(0) - \lambda_{r-2}[\sigma_c^{(r-1)} - K_c T F^{r-1} e^{Ft} \sigma_c^{(r-j)}(0)] - \ldots - \lambda_0[\dot{\sigma}_c(x, t) - K_c T F e^{Ft} \sigma_c^{(r-j)}(0)]|,
\]  (37)

with \( \eta_1 := \eta_\Omega, \eta_2 := \eta_d \).

For the IPMS motor, as the uncertainties are bounded, then there exist gains \( \alpha_\Omega \) and \( \alpha_d \) such that

\[
\dot{S}_\Omega S_\Omega \leq -\eta_\Omega |S_\Omega| \text{ and } \dot{S}_{i_d} S_{i_d} \leq -\eta_d |S_{i_d}|.
\]

Then, the finite time convergence of tracking errors is ensured.
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5. Conclusions
Principle and objectives

Fied Oriented Control strategy: FOC [Blaschke, 72]

Make electromagnetic torque proportional to the stator current ⇒ use the IM such as a Direct Motor.
Principle and objectives

Fied Oriented Control strategy: FOC [Blaschke, 72]

Make electromagnetic torque proportional to the stator current ⇒ use the IM such as a Direct Motor.

Control law
Principle and objectives

**Fied Oriented Control strategy: FOC** [Blaschke, 72]

Make **electromagnetic torque proportional** to the stator current ⇒ use the IM such as a Direct Motor.

**Control law**

- High Order Sliding Mode controller to achieve speed tracking in finite time
Principle and objectives

Fied Oriented Control strategy: FOC [Blaschke, 72]

Make electromagnetic torque proportional to the stator current ⇒ use the IM such as a Direct Motor.

Control law

- High Order Sliding Mode controller to achieve speed tracking in finite time
- High Order Sliding Mode controller to achieve flux tracking in finite time
Stator pulsation $\omega_s$

For FOC, one needs $\omega_s = p\Omega + a \frac{M_{sr}}{\phi_{rd}} i_{sq}$.

One defines

$$\tilde{\omega}_s = p\hat{\Omega} + a \frac{M_{sr}}{\hat{\phi}_{rd}} i_{sq} - \frac{(i_{sq} - \hat{i}_{sq})}{\beta_1 \hat{\phi}_{rd}} k_{\omega_s}$$

with $\tilde{\omega}_s$ an estimated stator frequency, $\beta_1 = \frac{M_{sr}}{\sigma L_s L_r}$ and $k_{\omega_s} > 0$. 
HOSM controller design (6)

Sliding vector

\[ s = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} \hat{\phi}_{rd} - \phi^* \\ \hat{\Omega} - \Omega^* \end{bmatrix} \]  \hspace{1cm} (38)

Relative degree of \( \sigma_1 \) and \( \sigma_2 = 2 \)

In order to limit chattering effect, design of a 3\( ^{rd} \) order sliding mode controller

The control input \( u \) is composed by a “linearizing-decoupling” term (in nominal case) and a term \( \nu \) defined such that

\[ \dot{\nu} = \begin{bmatrix} \dot{\nu}_1 \\ \dot{\nu}_2 \end{bmatrix} = \begin{bmatrix} -\alpha_\phi \text{sign}(S_\phi) \\ -\alpha_\Omega \text{sign}(S_\Omega) \end{bmatrix} \]
HOSM controller design (7)

The switching vector reads as
HOSM controller design (7)

The switching vector reads as

For \( t \leq t_F \).

\[
S_{\phi} = \ddot{\sigma}_1 - \chi_{\phi}, \\
S_{\Omega} = \ddot{\sigma}_2 - \chi_{\Omega}
\]

where

\[
\begin{align*}
\chi_{\phi} &= K_{\phi} F^2 e^{Ft} T \sigma_1(0) - 2\zeta_{\phi} \omega_{n\phi} (\dot{\sigma}_1 - K_{\phi} F e^{Ft} T \sigma_1(0)) - \omega_{n\phi}^2 (\sigma_1 - K_{\phi} e^{Ft} T \sigma_1(0)) \\
\chi_{\Omega} &= K_{\Omega} F^2 e^{Ft} T \sigma_2(0) - 2\zeta_{\Omega} \omega_{n\Omega} (\dot{\sigma}_2 - K_{\Omega} F e^{Ft} T \sigma_2(0)) - \omega_{n\Omega}^2 (\sigma_2 - K_{\Omega} e^{Ft} T \sigma_2(0))
\end{align*}
\]
HOSM controller design (7)

The switching vector reads as

- For $t \leq t_F$.

  \[
  S_\phi = \ddot{\sigma}_1 - \chi_\phi, \\
  S_\Omega = \ddot{\sigma}_2 - \chi_\Omega
  \]

  where

  \[
  \begin{align*}
  \chi_\phi &= K_\phi F^2 e^{Ft} T \sigma_1(0) - 2 \zeta_\phi \omega_{n\phi}(\dot{\sigma}_1 - K_\phi F e^{Ft} T \sigma_1(0)) - \omega_{n\phi}^2 (\sigma_1 - K_\phi e^{Ft} T \sigma_1(0)) \\
  \chi_\Omega &= K_\Omega F^2 e^{Ft} T \sigma_2(0) - 2 \zeta_\Omega \omega_{n\Omega}(\dot{\sigma}_2 - K_\Omega F e^{Ft} T \sigma_2(0)) - \omega_{n\Omega}^2 (\sigma_2 - K_\Omega e^{Ft} T \sigma_2(0))
  \end{align*}
  \]

- For $t > t_F$.

  \[
  S_\phi = \ddot{\sigma}_1 + 2 \zeta_\phi \omega_{n\phi} \dot{\sigma}_1 + \omega_{n\phi}^2 \sigma_1 \\
  S_\Omega = \ddot{\sigma}_2 + 2 \zeta_\Omega \omega_{n\Omega} \dot{\sigma}_2 + \omega_{n\Omega}^2 \sigma_2
  \]
HOSM controller design (7)

The switching vector reads as

- For \( t \leq t_F \).

\[
S_\phi = \ddot{\sigma}_1 - \chi_\phi, \\
S_\Omega = \ddot{\sigma}_2 - \chi_\Omega
\]

where

\[
\begin{align*}
\chi_\phi &= K_\phi F^2 e^{ft} T \sigma_1(0) - 2 \zeta_\phi \omega_n \dot{\sigma}_1 - K_\phi F e^{ft} T \sigma_1(0) - \omega^2_n (\sigma_1 - K_\phi e^{ft} T \sigma_1(0)) \\
\chi_\Omega &= K_\Omega F^2 e^{ft} T \sigma_2(0) - 2 \zeta_\Omega \omega_n \dot{\sigma}_2 - K_\Omega F e^{ft} T \sigma_2(0) - \omega^2_n (\sigma_2 - K_\Omega e^{ft} T \sigma_2(0))
\end{align*}
\]

- For \( t > t_F \).

\[
S_\phi = \ddot{\sigma}_1 + 2 \zeta_\phi \omega_n \dot{\sigma}_1 + \omega^2_n \sigma_1 \\
S_\Omega = \ddot{\sigma}_2 + 2 \zeta_\Omega \omega_n \dot{\sigma}_2 + \omega^2_n \sigma_2
\]

Stability of Controller-Observer scheme

Stability proof formally established in the paper.
Outline

1 Introduction

2 Observer without mechanical sensor
   2.a Observer design for non linear systems
   2.b Interior Permanent Magnet Synchronous Motor synchronous motor case (IPMSM)
      Model in (d-q) axes
      Observer stability analysis
   2.c Induction Motor case (IM)
      Induction motor model in a $dq$ rotating frame
      Adaptive Interconnected observer design
      Stability analysis of observer

3 High Order Sliding Control (HOSMC)
   3.a HOSMC design
   3.b IPMSM case
   3.c IM case

4 Sensorless control results
   4.a IPMSM case
      Benchmark trajectories
      Simulation results
   4.b IM case
      Benchmark trajectories

5 Conclusions
## Motor parameters

**Table:** Motor parameters (nominal values)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current</td>
<td>6A</td>
</tr>
<tr>
<td>Speed</td>
<td>3000 rpm</td>
</tr>
<tr>
<td>$R_s$</td>
<td>3.25 Ω</td>
</tr>
<tr>
<td>$L_d$</td>
<td>18 mH</td>
</tr>
<tr>
<td>$J$</td>
<td>0.00417 kg.m²</td>
</tr>
<tr>
<td>Torque</td>
<td>5.3 Nm</td>
</tr>
<tr>
<td>$\psi_f$</td>
<td>0.341 Wb</td>
</tr>
<tr>
<td>$p$</td>
<td>3</td>
</tr>
<tr>
<td>$L_q$</td>
<td>34 mH</td>
</tr>
<tr>
<td>$f_v$</td>
<td>0.0034 kg.m²s⁻¹</td>
</tr>
</tbody>
</table>
Benchmark (defined within the framework of the CNRS french workgroup CSE)

![Graph showing speed and load torque over time](image)

Figure: 3. Sensorless industrial benchmark. **a.** Speed reference, **b.** Load torque

Electric System Control group SEEDS/MACS
http://www2.irccyn.ec-nantes.fr/CSE/
Figure: 4. **a. Nominal case** Observed speed, Measured speed  **b. Observation error**
Figure : 5. **a. Nominal case** Estimated resistance, Resistance **b.** Estimation error
Figure: 6. **Nominal case, a.** Observed load torque, Measured load torque **b.** Observation error
Figure: 7. **Nominal case** Observed rotor position, Measured rotor position
Figure : 8. **Robustness test**, t=0s: +35% $R_s$; t=2s: +50%$R_s$  
**a.** Estimated resistance, Resistance  
**b.** Estimation error
Figure 9. **Robustness test**, $t=0s$: $+35\% \, R_s$; $t=2s$: $+50\% R_s$, a. Observed speed, Measured speed b. Observation error.
Figure : 10. Robustness test, \( t=0 \text{s}: -12\% \); \( t=2 \text{s}: -50\% R_s \), a. Estimated resistance, Resistance b. Estimation error
Figure: 11. **Robustness test, t=0s: -12%; t=2s: -50%** $R_s$, a. Observed speed, Measured speed b. Observation error
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Trajectories

Figure: 12. Benchmark: a- speed ref.: $\Omega^*$ (rd/s), b- load torque ref.: $T_l^*$ (N.m), c- flux ref.: $\phi^*$ (Wb)
Trajectories

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- Low speed + nominal load torque,
Sensorless control results  4.b IM case

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- Very low speed (unobservables conditions) + nominal load torque,
- Robustness tests.
Sensorless control results

4. b IM case
Finite time convergence

Figure: 13. **Top.** Measured speed $\Omega$ (rad) and its estimated (rad) versus time (sec.). **Middle.** Reference flux (Wb) and its estimated (Wb) versus time (sec.). **Bottom.** Flux tracking error (Wb) versus time (sec.).
Finite time convergence

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Sensorless control results

### Finite time convergence

![Graphs showing measured speed, reference flux, and flux tracking error over time.](image)

**Figure:** 13. **Top.** Measured speed $\Omega$ (rad) and its estimated (rad) versus time (sec.). **Middle.** Reference flux (Wb) and its estimated (Wb) versus time (sec.). **Bottom.** Flux tracking error (Wb) versus time (sec.).

### Nominal case

![Graphs showing measured and estimated values in nominal case.](image)

**Figure:** 14. Experimental result in nominal case: a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.
Sensorless control results

4.b IM case
Figure: 15. Experimental result with rotor resistance variation (+50%): a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.
Figure: 15. Experimental result with rotor resistance variation (+50%) : a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.
Sensorless control results

4.b IM case

Figure 15. Experimental result with rotor resistance variation (+50\%): a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.

Figure 16. Experimental result with rotor resistance variation (-50\%): a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.
Figure: 17. Experimental result with rotor self-inductance variation (+10%): a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.
Figure : 17. Experimental result with rotor self-inductance variation (+10%) : a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.
Sensorless control results  

4.b IM case

(+10% Lr)

Figure : 17. Experimental result with rotor self-inductance variation (+10%) : a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.

(+10% Ls)

Figure : 18. Experimental result with stator self-inductance variation (+10%) : a, c: measured speed and load torque, e: reference flux, b, d, f, g: estimated speed, load torque, flux and stator resistance, h, i, j: speed, load torque and flux estimation error.
Conclusions
Conclusions

- Design of an adaptive interconnected high-gain observers for the estimation of the magnetic (fluxes), mechanical (speed, load torque) variables, stator resistance value, ...
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- Validation on “Sensorless Control Benchmarks” from french research group of the CNRS Control of Electric Systems, http://www2.irccyn.ec-nantes.fr/CSE/.
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- Robustness tests to verify the good performance of ”Observer+controller” schemes.